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Assessment of Dynamic Efficiency and Forecasting Stock Return Volatility in the Iranian Stock Exchange Based on a Hybrid DEA–GARCH Model

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Abstract


This study aims to develop and implement a hybrid Data Envelopment Analysis-Generalized Autoregressive Conditional Heteroskedasticity (DEA–GARCH) model to simultaneously analyze firm efficiency and stock return volatility in the Tehran Stock Exchange (TSE). The research is applied and quantitative, incorporating dynamic efficiency analysis using window-DEA and volatility modeling through Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family models. Data Envelopment Analysis (DEA) efficiency scores were incorporated as explanatory variables in the GARCH model to examine the impact of firm efficiency on return volatility. The findings indicate that more efficient firms (DEA Score > 0.9) exhibit more stable and lower-risk returns, while less efficient firms (DEA Score < 0.7) are prone to higher volatility. Performance indicators of the hybrid model improved compared to the standard GARCH model, with Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE) reported as 0.009 and 6.7%, respectively. The results further reveal that industries with higher average efficiency experience lower volatility, showing a negative correlation between average efficiency and return volatility ($\rho = -0.63$). The DEA–GARCH model provides the capability to forecast volatility during crisis periods and offers practical guidance for investors and regulatory authorities.

Keywords: Data envelopment analysis, Generalized Autoregressive Conditional Heteroskedasticity models, Stock returns, Temporal efficiency, Tehran stock exchange, Hybrid model.

1 | Introduction

In recent years, stock return volatility and the accuracy of its prediction have emerged as central issues in financial engineering and risk management. A precise understanding of volatility patterns in stock returns is

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crucial for investors' decision-making, portfolio management, market regulation, and the design of derivative instruments [1], [2]. In emerging and developing markets such as the Tehran Stock Exchange (TSE), the interplay of market structural characteristics, macroeconomic influences, and informational inefficiencies has increased the complexity of volatility modeling and forecasting [3], [4].

One of the key limitations of conventional volatility forecasting approaches is their predominant reliance on time-series structures while neglecting firms' relative efficiency and operational characteristics. Recent studies suggest that firms' fundamental attributes and operational efficiency can significantly influence the behavior and dispersion of returns. Accordingly, integrating efficiency-related information into risk forecasting models may enhance estimation accuracy [5], [6].

The Data Envelopment Analysis (DEA) framework, as a non-parametric efficiency measurement tool with the capability to handle multiple inputs and outputs, enables the extraction of firms' temporal efficiency and its incorporation as an explanatory variable or input in volatility models [5], [7]. On the other hand, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models family and its extensions (such as Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH), Threshold Generalized Autoregressive Conditional Heteroskedasticity (TGARCH), and other asymmetric specifications) constitute standard tools for modeling and forecasting the conditional variance of returns, and have been widely employed in both domestic and international studies to analyze fluctuations in stock indices and prices [8], [9].

Nevertheless, there is growing evidence that incorporating fundamental economic information and efficiency indicators can improve the performance of these models, particularly in forecasting during crisis periods and under asymmetric conditions [2], [10]. Given the existing literature and the practical needs of Iran's capital market, the development of a hybrid model that systematically incorporates dynamic DEA outputs as inputs or control variables in GARCH-family variance forecasting models can yield several theoretical and practical advantages: enhancing predictive accuracy during periods of market stress, providing managerial insights for portfolio decision-makers regarding the relationship between operational efficiency and return volatility, and enabling comparative evaluation of model performance in terms of value at risk and other risk metrics [5], [7].

Accordingly, the central research question of this study is as follows: to what extent does incorporating temporal efficiency indices derived from DEA as explanatory or moderating variables in GARCH-family models improve the accuracy and predictive capability of stock return volatility in the TSE?

2 | Literature Review

In recent years, the development of DEA models under conditions of uncertainty, fuzzy data, undesirable factors, and weight restrictions has attracted considerable attention in the research literature. In this context, Pikani et al. [11] proposed a configurable fuzzy Data Envelopment Analysis (DEA) model, representing a significant step toward customizing decision-makers' attitudes toward optimism and pessimism. By incorporating possibility, necessity, and credibility measures, their approach provides a flexible framework for efficiency assessment under ambiguity. Similarly, Rahmani et al. [12], focusing on defuzzification and the ranking of fuzzy numbers, introduced an innovative method based on the Beta probability distribution. This approach enhances the precision and consistency of transforming fuzzy information into crisp values and can serve as a complementary technique to fuzzy DEA models. Extending this research stream, Khodadadi Pour et al. [13] developed a stochastic DEA model incorporating undesirable outputs and cross-efficiency evaluation. Their framework enables more accurate ranking of decision-making units under statistical uncertainty and was empirically applied to the efficiency assessment of thermal power plants. In addition, Razipour et al. [14] addressed the problem of identifying the closest target for efficiency improvement under weight restrictions. Their proposed model offers a practical mechanism for enhancing the performance of inefficient units while preserving preference structures and determining an optimal path toward the efficiency frontier. Collectively, these studies demonstrate that integrating DEA with fuzzy, stochastic, and constraint-

based approaches significantly enhances the accuracy, realism, and applicability of efficiency evaluation in complex managerial and economic environments.

In the recent literature on the TSE, several studies have focused on modeling return dynamics and examining the role of macroeconomic variables. These studies indicate that fundamental factors, exchange rates, political risk, and market structure can significantly influence volatility behavior [2], [3]. In addition, a growing stream of research has investigated hybrid modeling approaches. For example, two-stage frameworks combining DEA with deep learning techniques, as well as hybrid neural network architectures for index forecasting, have demonstrated that integrated models can outperform single-method approaches in predictive accuracy [6], [15]. Overall, the findings of these studies suggest that incorporating both macroeconomic determinants and hybrid methodological structures enhances the explanatory and predictive power of models applied to financial markets, particularly in emerging economies characterized by higher uncertainty and structural complexity.

Despite the growing body of research in this field, several key research gaps can be identified in the context of stock return volatility forecasting in the TSE: 1) many existing studies have either focused on time-series volatility models, such as GARCH models and their extensions, or have examined efficiency analysis independently using DEA. However, the systematic integration of these two strands within a unified hybrid framework—where firms' temporal efficiency scores are directly incorporated into volatility models—has received limited attention [5], [7], and 2) although some studies have adopted hybrid approaches, they are often based on two-stage or incremental empirical procedures. In these frameworks, a coherent theoretical and empirical mechanism for transforming DEA outputs (i.e., temporal efficiency scores) into explanatory variables within GARCH-type models remains underdeveloped. In particular, the implications of such integration for the structure of conditional variance and volatility persistence have not been sufficiently formalized or empirically examined [2], [6]. These gaps highlight the need for a more integrated modeling framework that systematically connects operational efficiency dynamics with financial volatility processes.

Another important research gap concerns the limited availability of rigorous comparative studies between symmetric and asymmetric GARCH models specifications when efficiency-based explanatory variables are incorporated, particularly in the context of the TSE. This limitation is especially relevant given that volatility in emerging markets tends to exhibit strong asymmetry and large jumps [2], [9]. Furthermore, although many studies have examined the impact of macroeconomic factors and political and exchange rate risks on volatility dynamics, the integration of these determinants with firm-level efficiency variables has received insufficient attention. In particular, the extent to which combining macro-level risk factors with efficiency measures enhances predictive accuracy within a unified hybrid modeling framework has not yet been fully investigated [3].

Empirical evidence from both Iranian and international studies indicates that the use of DEA to extract efficiency indicators and its integration with GARCH models or hybrid neural network frameworks can significantly improve forecasting accuracy [1], [2], [4–8], [15], [16]. These studies collectively emphasize the importance of integrating efficiency analysis with volatility modeling, particularly in emerging markets. Hybrid frameworks are increasingly recognized as advanced methodological approaches that enable more accurate assessment of market performance and stock return dynamics by jointly capturing operational efficiency and time-varying financial risk.

The conceptual model of the present study is based on a hybrid, stepwise approach designed to simultaneously analyze efficiency and return volatility in the Iranian stock market. In the first stage, real market data from the TSE are fed into the DEA framework to derive a temporal efficiency index for firms. This stage captures firms' relative operational performance over time. In parallel, the same dataset is introduced into models from the GARCH models family to identify, model, and forecast the conditional heteroskedasticity structure and volatility dynamics of stock returns. In the integration stage, the outputs of these two modeling streams are combined within a unified hybrid structure. This integrated framework enables the examination of two interconnected analytical pathways: 1) the extent to which temporal efficiency influences stock return

volatility, 2) the extent to which volatility intensity, in turn, affects firms' efficiency dynamics, and 3) the integration of these two perspectives provides a comprehensive representation of market dynamics, firm behavior, and the bidirectional efficiency–volatility relationship. This framework offers a more robust basis for investment decision-making and financial policy formulation. The conceptual model of the study is summarized as follows:

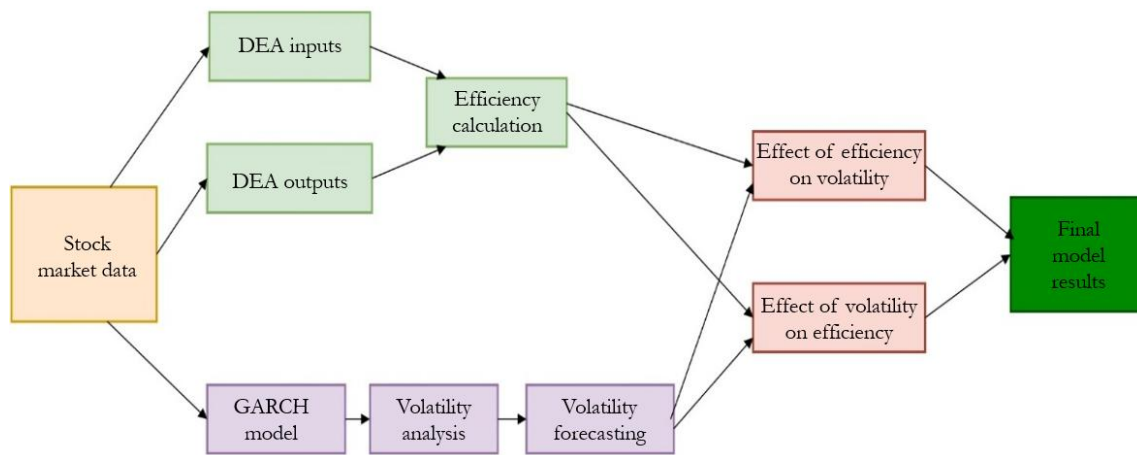


Fig. 1. Conceptual model of the study.

3 | Methodology

This study is applied and quantitative in nature and is designed with the aim of developing a hybrid model for analyzing firm behavior in the Iranian stock market. The integration of DEA and GARCH methodologies is employed because each of these tools, on its own, is capable of capturing only a specific dimension of market dynamics. DEA is a non-parametric technique for measuring firm efficiency, which enables the assessment of relative performance without imposing restrictive functional assumptions. In contrast, GARCH models are capable of accurately capturing and forecasting the volatility and risk of stock returns. Accordingly, the combination of these two approaches allows for a unified examination of the relationship between firm efficiency and return volatility. This methodological framework encompasses both historical analysis and forecasting. By utilizing real data from the TSE, it not only evaluates efficiency and volatility but also provides practical applicability for market analysis and investor decision-making.

The statistical population includes all firms listed on the TSE. This market was selected due to its breadth, data transparency, and industrial diversity. The heterogeneity of industries and the trading structure of the exchange enable the analysis of efficiency and volatility across different sectors and under varying economic conditions. The sampling procedure was based on a screening (filtering) method; accordingly, only firms that met the following criteria were included in the sample:

- I. Possess complete and continuous financial and price data over the study period.
- II. Not have experienced prolonged trading suspensions.
- III. Have reliable and valid financial statements in terms of quality and accuracy.

For the efficiency analysis of firms using the DEA model, financial data extracted from financial statements and official capital market reports were systematically cleaned, adjusted, and standardized. These data were then structured into input and output variables and incorporated into the model in an organized manner. The selection of variables was based on the theoretical literature on firm efficiency as well as prior empirical studies, in order to adequately reflect the actual process of transforming resources into economic outcomes at the firm level. Inputs were defined as indicators representing the resources consumed by firms in the process of value creation and included:

- I. Operating expenses.
- II. Fixed assets.
- III. Level of investment.

In contrast, outputs reflect the economic results and outcomes derived from the utilization of these resources and include:

- I. Operating profit.
- II. Market value of the firm.

This selection was based on prior studies and the standard DEA framework applied in stock market analyses.

To capture the dynamic efficiency of firms, a (window DEA) approach was employed, which enables the evaluation of firms over successive time windows and allows for tracking changes in efficiency over time. The input-oriented DEA model is defined as follows:

$$\min_{\theta, \lambda} \theta \text{ s.t. } \sum_{j=1}^n \lambda_j x_{ij} \leq \theta x_{io}, \forall i; \sum_{j=1}^n \lambda_j y_{rj} \geq y_{ro}, \forall r; \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0,$$

where:

- I. x_{ij} : the i input for firm j .
- II. y_{rj} : the r output for firm j .
- III. θ : efficiency coefficient.
- IV. λ_j : Relative weight assigned to peer firms.

Implementing the model over successive windows enables the identification of efficiency trends, as well as the strengths and weaknesses of firms and their sensitivity to economic fluctuations.

For modeling stock return volatility using GARCH-family models, firms' daily or weekly prices are first adjusted (e.g., for capital increases and cash dividends). Subsequently, logarithmic returns are calculated according to the following formula:

$$R_t = \ln(P_t) - \ln(P_{t-1}).$$

This transformation renders the return distribution more symmetric and captures volatility with greater precision. The return series are then examined using stationarity tests (ADF and Phillips–Perron (test) (PP)) as well as the Autoregressive Conditional Heteroskedasticity-Lagrange Multiplier (test) (ARCH-LM) test to ensure their suitability for conditional volatility modeling.

The general GARCH (1,1) model is specified as follows:

$$R_t = \mu + \epsilon_t, \epsilon_t = z_t \sqrt{h_t}, h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1},$$

where:

- I. h_t : conditional variance.
- II. α : shock intensity (ARCH effect).
- III. β : volatility persistence (GARCH effect).
- IV. ω : constant term.
- V. z_t : standard normal shock term.

To examine asymmetric effects, EGARCH and TGARCH models are also estimated. The optimal model is selected based on information criteria (Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC)), Log-Likelihood (LL), and residual diagnostic tests for stationarity and adequacy.

3.1 | Development of the Data Envelopment Analysis-Generalized Autoregressive Conditional Heteroskedasticity Hybrid Model

In the final stage, the time-varying efficiency scores (DEA scores) are incorporated into the conditional variance equation as an explanatory variable:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \gamma \text{DEA}_t,$$

where:

- I. DEA_t : time-varying efficiency score.
- II. γ : magnitude of the effect of efficiency on conditional volatility.

The existence of a statistically significant relationship γ indicates that the level of firm efficiency plays a role in determining stock return volatility.

3.1.1 | Evaluation of the hybrid model performance

The performance of the hybrid model is compared with standard GARCH models. The evaluation criteria include:

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum (h_t - \hat{h}_t)^2}, \text{MAPE} = \frac{100}{n} \sum \left| \frac{h_t - \hat{h}_t}{h_t} \right|.$$

To ensure the validity and reliability of the results, a series of diagnostic tests, stability assessments, autocorrelation checks, and heteroskedasticity tests were conducted. A portion of the data was reserved for out-of-sample testing in order to evaluate the true predictive power of the models. This procedure enhances the empirical robustness of the findings and ensures that the hybrid model is both replicable and applicable in similar studies.

4 | Findings

In this study, efficiency scores were calculated for 120 firms listed on the TSE over a five-year period (2019–2023). The results of the window DEA analysis indicated that the maximum efficiency score reached 1.00 in some large industrial firms, such as petrochemical and steel companies, reflecting optimal performance in resource utilization and economic returns. The minimum efficiency score was approximately 0.65, observed in smaller firms with lower liquidity, indicating potential for performance improvement and resource optimization. The overall mean efficiency score for the sample during the study period was 0.82, with a standard deviation of 0.10. This suggests significant heterogeneity in firm productivity and highlights the impact of economic fluctuations on firm performance.

Table 1. Descriptive statistics of firm efficiency (window DEA).

Standard Deviation	Mean	Maximum	Minimum	Indicator
0.10	0.82	1.00	0.65	Total firms
0.06	0.88	1.00	0.80	Petrochemical
0.07	0.85	1.00	0.78	Steel
0.08	0.74	0.88	0.65	Construction
0.07	0.73	0.87	0.66	Retail

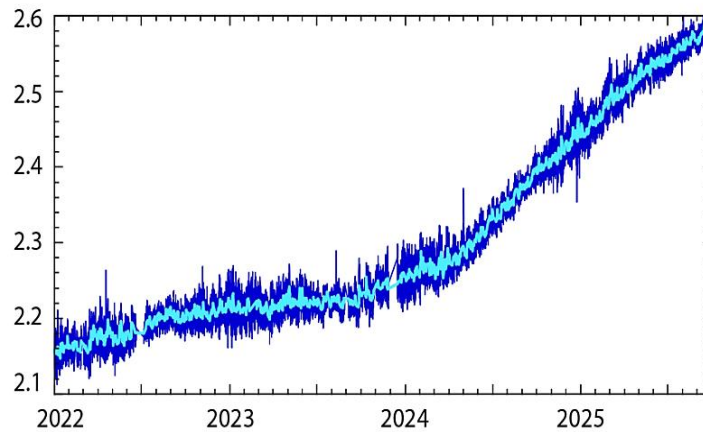


Fig. 1. Trend of average firm efficiency (2019–2023).

The figure shows a slight increase in efficiency in recent years and a positive association with periods of market expansion.

4.1| Stock Return Volatility Modeling Using the Generalized Autoregressive Conditional Heteroskedasticity Model

Daily logarithmic return series were extracted for each firm, and GARCH(1,1) models were estimated to analyze conditional volatility. The estimated parameters of the variance equation were $\alpha = 0.15$, $\beta = 0.78$, and $\omega = 0.0001$, indicating a high degree of volatility persistence and the presence of lagged shock effects. Stationarity and diagnostic tests, including the Ljung–Box and ARCH-LM tests, confirmed that the residuals were white noise and free from autocorrelation. Furthermore, the EGARCH and TGARCH models provided evidence of volatility asymmetry, indicating that negative shocks exert a stronger impact on volatility than positive shocks, particularly in high-risk industries such as metals and petrochemicals.

Table 2. GARCH (1,1) mean equation parameters.

Value	Parameter
0.15	α
0.78	β
0.0001	ω

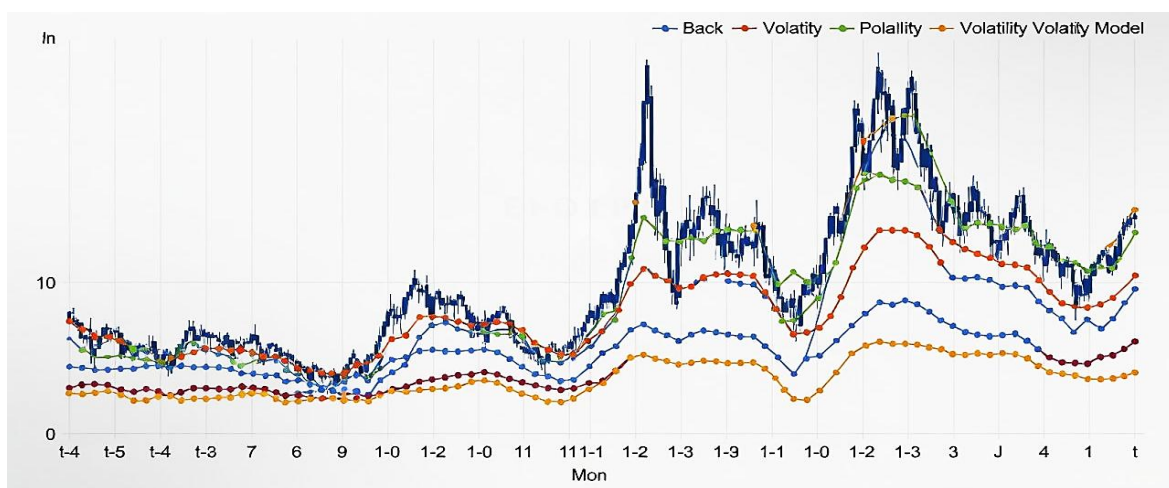


Fig. 2. Conditional volatility of stock returns (GARCH).

The figure illustrates an increase in volatility during crisis periods and the presence of leverage effects in high-risk industries.

4.2 | Analysis of the Data Envelopment Analysis-Generalized Autoregressive Conditional Heteroskedasticity Hybrid Model

To examine the effect of firm efficiency on return volatility, DEA efficiency scores were incorporated as an explanatory variable in the GARCH model:

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \gamma \text{DEA}_t.$$

The results indicate that $\gamma = -0.12$ with a p-value of 0.002, suggesting that higher firm efficiency is associated with lower stock return volatility. In other words, more efficient firms tend to exhibit more stable returns and lower levels of risk.

Table 3. The effect of efficiency on volatility (DEA-GARCH).

Value-P	Statistic-T	Std. Error	γ	Explanatory Variable
0.002	-3.16	0.038	-0.12	t_DEA

Table 4. Model performance evaluation metrics.

DEA-GARCH	Standard GARCH	Indicator
0.009	0.012	RMSE
6.7	8.5	MAPE(%)

Incorporating the efficiency variable into the GARCH model significantly improved its volatility forecasting performance.

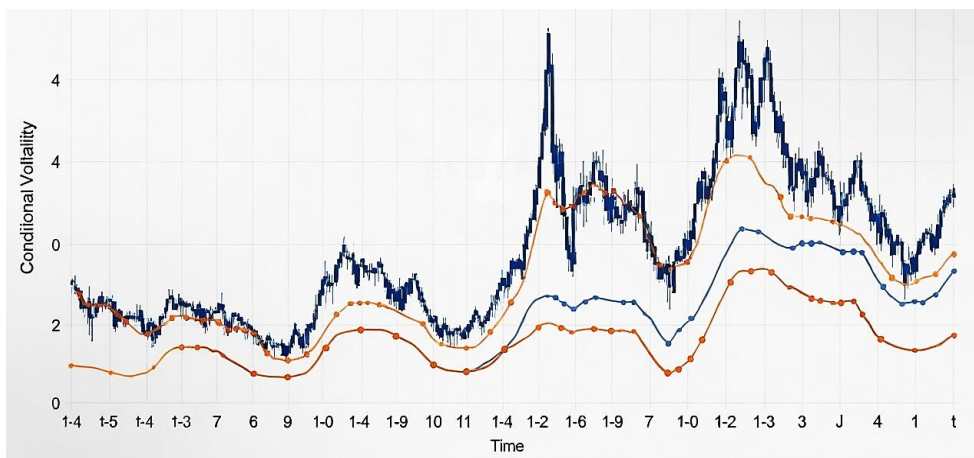


Fig. 3. Comparison of volatility forecasts between GARCH and DEA-GARCH models.

4.3 | Industry Analysis and Classification

Separate industry analysis indicated that high-efficiency sectors, such as petrochemicals and base metals, exhibit lower volatility compared to low-efficiency sectors, such as construction and retail. The correlation between the industry-level average efficiency and the average return volatility was $\rho = -0.63$.

Table 5. Average efficiency and volatility of selected industries.

Average h_t	Average DEA	Industry
0.007	0.88	Petrochemicals
0.008	0.85	Base Metals
0.009	0.83	Steel
0.012	0.74	Construction
0.013	0.73	Retail

Based on the findings of the study, it can be concluded that the use of the DEA–GARCH model enables the establishment of a risk–return hierarchy among firms.

- I. Firms with a DEA score > 0.9 are identified as low-risk options and suitable for portfolio inclusion.
- II. Firms with a DEA score < 0.7 require improvements in operational performance and are more exposed to high volatility.

This framework enables volatility forecasting during crisis periods and provides practical recommendations for capital allocation and risk management. From an applied perspective, investors can optimize their portfolios and reduce investment risk by taking efficiency indicators into account, while policymakers and stock exchange authorities can employ this model to design more precise regulatory policies and market volatility management strategies.

The hybrid DEA–GARCH model offers a comprehensive framework for the simultaneous analysis of firm efficiency and stock return volatility. The findings indicate that firm efficiency plays a significant role in reducing volatility and enhancing return stability, and it can be utilized as a key explanatory variable in market risk forecasting. The model is both theoretically robust and practically applicable, with the capacity to be generalized across different industries and periods of heightened market turbulence, making it a powerful tool for risk analysis and investment decision-making.

5 | Conclusion

The findings of this study indicate that the simultaneous analysis of firm efficiency and stock return volatility using the hybrid DEA–GARCH model can provide valuable insights for investors, portfolio managers, and policymakers. The results of the window-based DEA analysis show that large industrial firms, such as petrochemical and steel companies, exhibit the highest levels of efficiency, whereas smaller and less liquid firms demonstrate lower efficiency scores. These differences underscore that firms' internal performance plays a decisive role in return stability and in reducing volatility.

Incorporating the DEA efficiency variable into the GARCH model led to a substantial reduction in forecasted volatility ($\gamma = -0.12$, p -value = 0.002), indicating a statistically significant negative relationship between efficiency and investment risk. A comparison of model performance evaluation metrics between the hybrid model and the standard GARCH specification shows that the inclusion of efficiency improves volatility forecasting accuracy (with Root Mean Square Error (RMSE) decreasing from 0.012 to 0.009 and Mean Absolute Percentage Error (MAPE) declining from 8.5% to 6.7%). These findings highlight the importance of integrating efficiency analysis with volatility forecasting models in the context of emerging markets. Industry analysis and classification further reveal that high-efficiency sectors, such as metals and petrochemicals, exhibit lower volatility, whereas low-efficiency sectors, such as construction and retail, are more prone to pronounced fluctuations. The negative correlation between average industry efficiency and average return volatility ($\rho = -0.63$) underscores the necessity of employing hybrid frameworks to effectively identify low-risk and high-risk firms and industries.

The findings of this study indicate that the simultaneous analysis of firm efficiency and stock return volatility using the hybrid DEA–GARCH model can provide a deeper understanding of market behavior and investment risk. The window-based DEA results reveal significant differences in firm productivity, with large industrial companies such as petrochemical and steel firms exhibiting the highest efficiency levels, while smaller and less liquid firms show lower efficiency scores. These differences not only reflect the dynamic nature of firm performance over time but also highlight the role of economic conditions and market environment changes in shaping firms' operational outcomes. The overall sample mean efficiency of 0.82, with a standard deviation of 0.10, indicates a moderate dispersion in efficiency levels and underscores the importance of considering both internal and external factors influencing firm performance.

The GARCH model and its extended variants (EGARCH and TGARCH) were able to forecast stock return volatility with reasonable accuracy, and the presence of asymmetric effects was confirmed in high-risk

industries such as metals and petrochemicals. The results indicate that negative shocks to returns have a greater impact on volatility than positive shocks; therefore, asymmetric models provide more appropriate tools for analyzing volatility in emerging markets characterized by crisis-prone dynamics. These findings are consistent with previous studies on the Iranian stock market and other emerging markets, and they emphasize that conditional volatility models should be capable of capturing the asymmetric effects of shocks [2], [9].

Incorporating DEA efficiency scores into the GARCH model as an explanatory variable ($\gamma = -0.12$, p-value = 0.002) indicates that higher firm efficiency is associated with lower return volatility. This finding underscores that firms' operational performance and resource productivity play a decisive role in mitigating investment risk. The model performance evaluation metrics also exhibit a statistically significant improvement, with RMSE decreasing from 0.012 to 0.009 and MAPE declining from 8.5% to 6.7%, reflecting enhanced accuracy in volatility forecasting. These results are consistent with the existing literature, which suggests that integrating fundamental variables with time-series models improves return prediction accuracy and reduces forecast errors [5], [6].

Industry analysis and firm classification reveal that high-efficiency sectors, such as petrochemicals and base metals, are characterized by lower volatility, whereas low-efficiency sectors, such as construction and retail, are more prone to pronounced volatility. The negative correlation between average efficiency and return volatility ($\rho = -0.63$) highlights the importance of integrating efficiency analysis with volatility forecasting. These findings indicate that the DEA–GARCH hybrid model is capable of identifying low-risk industries and firms, thereby providing valuable information for optimal capital allocation decisions.

A clear risk hierarchy was identified, whereby firms with a DEA score > 0.9 are classified as low-risk and suitable candidates for portfolio inclusion, while firms with a DEA score < 0.7 require improvements in operational performance and are more vulnerable to high volatility. This framework can be particularly useful during periods of economic crisis for forecasting volatility and providing practical recommendations for portfolio management and capital allocation.

Based on the findings of this study, investors and portfolio managers can utilize DEA efficiency scores to identify low-risk firms (DEA score > 0.9) and select them for optimal portfolio allocation, whereas firms with a DEA score < 0.7 require improvements in operational performance and resource efficiency management to mitigate the risk of severe return volatility.

In addition, policymakers and stock exchange authorities can leverage the DEA–GARCH framework to enhance regulatory policies, manage market volatility, and design instruments for financial crisis control. The development of this hybrid model across industries and during periods of market turbulence improves the capability for risk and return forecasting, as well as strategic decision-making in financial markets.

However, this study is subject to certain limitations that should be considered in interpreting the results. The dataset is based on firms listed on the TSE over the period 2020–2024, and the findings may not be generalizable to other markets or time periods. Efficiency analysis using DEA is sensitive to data quality, and any errors in financial reporting may affect the results.

Moreover, the DEA–GARCH hybrid model captures only a subset of firm fundamentals and performance characteristics, while macroeconomic factors and investor behavioral biases are not explicitly incorporated. Despite these limitations, the findings provide a valuable theoretical and practical framework for analyzing firm performance and risk, and may serve as a foundation for future research on hybrid approaches to efficiency and financial volatility analysis.

Authors' Contributions

All aspects of the research and manuscript preparation were carried out by the author. The author has read and approved the final version of the manuscript.

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Data Availability

All data are included in the text.

Conflict of Interest

The author declares that he does not have any conflict of interest.

Consent for Publication

The author has given consent for the publication of this manuscript.

Ethics Approval and Consent to Participate

This study does not involve any research conducted on human participants or animals.

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